

Chapter 2

Assignment 2 Solutions

2.5. a. Consider a fiber with a $100\text{ }\mu\text{m}$ core diameter and a $140\text{ }\mu\text{m}$ cladding diameter. If $n_1 = 1.48$ and $\Delta = 1\%$, calculate the V -parameter if the operating wavelength is 850 nm .

b. ... if the wavelength is 1300 nm ?

c. Find the value of V at a wavelength of 850 nm if the diameter of the core is $50\text{ }\mu\text{m}$?

Solution: a. We have $a = 50 \times 10^{-6}\text{ m}$, $n_1 = 1.48$, $\Delta = 0.01$, and $\lambda = 850 \times 10^{-9}$, so

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi(50 \times 10^{-6})}{850 \times 10^{-9}} (1.48) \sqrt{2(0.01)} = 77.4. \quad (2.1)$$

b. We change λ to 1300×10^{-9} , so

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi(50 \times 10^{-6})}{1300 \times 10^{-9}} (1.48) \sqrt{2(0.01)} = 50.6. \quad (2.2)$$

c. We change λ back to 850×10^{-9} and a to 25×10^{-6} , so

$$V = \frac{2\pi a}{\lambda} n_1 \sqrt{2\Delta} = \frac{2\pi(25 \times 10^{-6})}{850 \times 10^{-9}} (1.48) \sqrt{2(0.01)} = 38.7. \quad (2.3)$$

2.6. a. Calculate the number of modes for each case described in the previous problem.

b. Calculate the percentage of the optical power that is carried in the cladding for each case described in the prior problem.

Solution: a. The number of modes is given by

$$N \approx \frac{V^2}{2}. \quad (2.4)$$

For $V = 77.4$, $N \approx 3000$ (rounded to three significant figures); for $V = 50.6$, $N \approx 1280$; and for $V = 38.7$, $N \approx 749$.

b. The ratio of the power in the cladding to the power in the core is

$$\frac{P_{\text{cladding}}}{P_{\text{Total}}} \approx \frac{4}{3\sqrt{N}}. \quad (2.5)$$

So, for $N = 3000$, the ratio is 0.0244; for $N = 1280$, the ratio is 0.0373; for $N = 749$, the ratio is 0.0487.

2.10. a. Design a single-mode step-index fiber for operation at $1.3 \mu\text{m}$ with a fused silica core ($n_1 = 1.458$). Find n_2 and the diameter of the core.

b. Is the fiber still single-mode at 820 nm ? If not, how many modes are there?

Calculate the cutoff wavelength λ_c .

Solution: a. We want a value of V within the range $2.0 < V < 2.405$; I will arbitrarily pick $V = 2.3$. (Your choice of V might be different.) We know that

$$V = \frac{2\pi a}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{2\pi a \text{NA}}{\lambda}. \quad (2.6)$$

We have a free choice on either n_2 or a . I will pick $a = 4 \mu\text{m}$ to form an $8 \mu\text{m}$ core (which I know is approximately what is used to make commercially-available single-mode fibers). So,

$$V = \frac{2\pi a \text{NA}}{\lambda} \quad (2.7)$$

$$2.30 = \frac{2\pi(4 \times 10^{-6})(\text{NA})}{(1300 \times 10^{-9})}$$

$$\text{NA} = \frac{(2.3)(1300 \times 10^{-9})}{2\pi(4 \times 10^{-6})} = 0.1190. \quad (2.8)$$

We then find

$$\text{NA}^2 = n_1^2 - n_2^2 \quad (2.9)$$

$$n_2^2 = n_1^2 - \text{NA}^2 = (1.458)^2 - (0.119)^2 = 2.11$$

$$n_2 = \sqrt{2.11} = 1.453.$$

(This value of n_2 is reasonable because it lies within the standard range for glass from 1.45 to 1.50.) Checking the value of Δ to see if it is realistic (i.e., within the range $2\% > \Delta > 0.1\%$),

$$\Delta = \frac{n_1 - n_2}{n_1} = \frac{(1.458 - 1.453)}{1.458} = 0.00334 = 0.334\%. \quad (2.10)$$

The value of Δ checks out OK.

Note: In solving this problem, you *must* check the following calculated values for reasonableness, a ($10 \mu\text{m} > a > 4 \mu\text{m}$) and Δ ($2\% > \Delta > 0.1\%$).

b. We want to find the number of modes in the fiber at 820 nm.

$$\frac{V(820)}{V(1300)} = \frac{1300}{820} \quad (2.11)$$

so

$$V(820) = V(1300) \frac{1300}{820} = (2.30)(1.585) = 3.65. \quad (2.12)$$

We note that the fiber is *not* single mode at 820 nm. Looking at the mode diagram, we find that there are two modes in the fiber at 820 nm, the 01 mode and the 11 mode. (Note: Do *not* use the formula for N ; it is valid for large values of V only.)

c. We calculate the cutoff wavelength λ_{co} as

$$\lambda_{\text{co}} = \frac{2\pi a \text{NA}}{2.405} = \frac{2\pi(4 \times 10^{-6})(0.1190)}{(2.405)} = 1.244 \times 10^{-6} = 1.244 \mu\text{m}. \quad (2.13)$$

2.14. Design a single-mode fiber (with $V = 2.3$) for operation at $1.3 \mu\text{m}$ with a fused silica core ($n_1 = 1.458$) The numerical aperture of the fiber is to be 0.10.

a. Find the cladding index n_2 and the radius a of the fiber.

b. Calculate the approximate number of modes in your fiber for operation at 820 nm.

Solution: a. We can find n_2 from the desired value of NA, as

$$n_2 = \sqrt{n_1^2 - (\text{NA})^2} = \sqrt{(1.458)^2 - (0.1)^2} = 1.454. \quad (2.14)$$

We find the value of a from the desired value of V ,

$$a = \frac{\lambda V}{2\pi \text{NA}} = \frac{(1300 \times 10^{-9})(2.3)}{(2\pi)(0.10)} = 4.76 \times 10^{-6} = 4.76 \mu\text{m}. \quad (2.15)$$

The fiber will have a core diameter d ($= 2a$) of $9.52 \mu\text{m}$.

b. The number of modes N in this fiber at 820 nm is found from

$$V(820) = V(1300) \left(\frac{1300}{820} \right) = (2.3)(1.585) = 3.646. \quad (2.16)$$

From the mode diagram, we find that *two modes* propagate at this value of V . (Note: we cannot use our approximation formula for N since V is not large.)

2.15. Using a computer, plot the refractive index profile from n_1 to n_2 vs. radial position for $g = 1, 2, 4, 8$ and ∞ . Assume a core diameter of $50 \mu\text{m}$, $n_1 = 1.480$, and $\Delta = 1\%$.

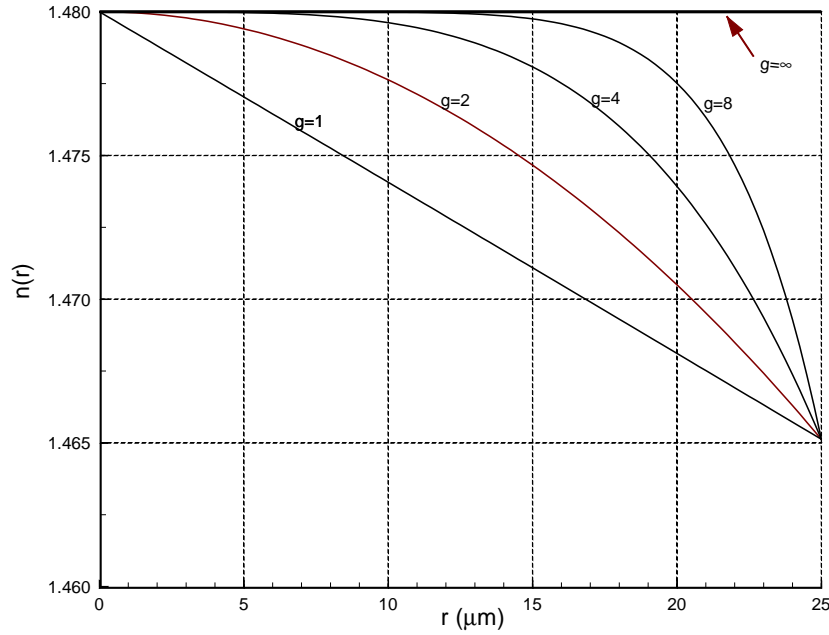


Figure 2.1: Solution of Problem 2.15

Solution: The equation for the index of refraction is

$$n(r) = n_1 \sqrt{1 - 2\Delta \left(\frac{r}{a}\right)^g} \quad \text{for } 0 < r \leq a. \quad (2.17)$$

The plot of this equation for $g = 1, 2, 4, 8$, and ∞ is shown in Fig. 2.1.

2.17. Prove that

$$\theta_{\max} = \sin^{-1} \sqrt{n_1^2 - n_2^2} \quad (2.18)$$

by applying Snell's Law at the fiber input face for the ray that meets the critical angle condition at the core-cladding interface. For generality, assume that the medium outside the fiber has an index n_0 and then check the equation for $n_0 = 1$.

Solution: The geometry of Fig. 2.2 applies. We see, by geometry, that

$$\theta_t = 90^\circ - \theta_c \quad (2.19)$$

and, from Snell's Law across the boundary, that

$$n_0 \sin \theta_i = n_1 \sin \theta_1. \quad (2.20)$$

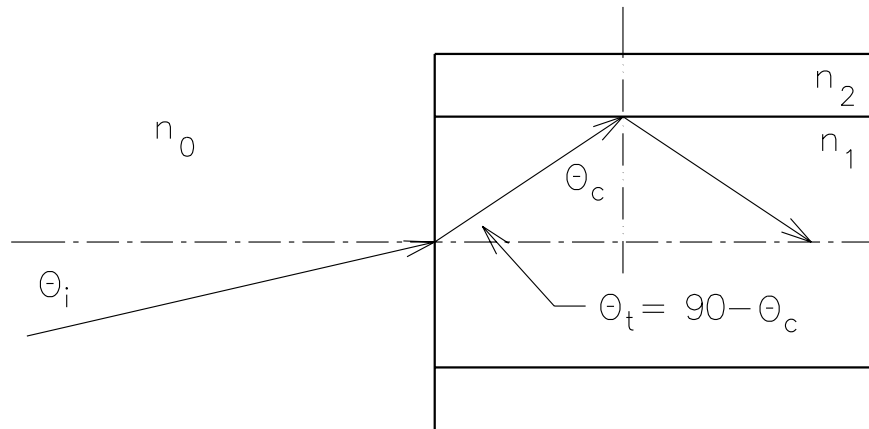


Figure 2.2: Geometry for Problem 2.17.

We also know that

$$\theta_c = \sin^{-1} \frac{n_2}{n_1}. \quad (2.21)$$

From the latter expression we know that

$$\sin(90^\circ - \theta_c) = \frac{\sqrt{n_1^2 - n_2^2}}{n_1}. \quad (2.22)$$

From the Snell's law relation, we have

$$\begin{aligned} \sin \theta_i &= \frac{n_1}{n_0} \frac{\sqrt{n_1^2 - n_2^2}}{n_1} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0} \\ &= \sqrt{n_1^2 - n_2^2} \quad (\text{for } n_0 = 1). \end{aligned} \quad (2.23)$$